Theoretical Condensed Matter Physics
Exercise 6

(Submission: 07.12.18, discussion: 10./12.12.18)

6.1 Thermoelectric effect (Seebeck effect) (10 points)

When a temperature difference $\Delta T = T_R - T_L$ is applied between the ends of a metallic wire, an electric voltage V_{th} is generated between the ends of the wire, see Fig. 1. This thermoelectric effect is called Seebeck effect after its discoverer. We want to understand the origin of this effect and its dependence on the wire properties.
The electrons in the wire are assumed to be in local equilibrium, i.e., *not* in the quantum regime (compare problem 5.1), and the temperature T is well defined at any point of the wire. A voltage drop, if it exists, is the difference between the chemical potentials, $V_{th} = (\mu(T_R) - \mu(T_L))/e =$.

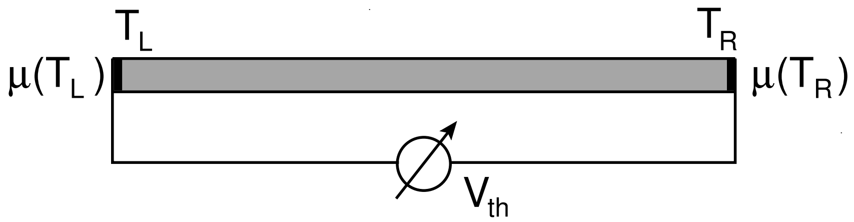

Figure 1: Thermoelectric effect: A temperature difference applied to a metal wire generates a thermovoltage $V_{th} = (\mu(T_R) - \mu(T_L))/e$.

(a) Express the total particle density ρ in terms of the density of states $N(\varepsilon)$ and the equilibrium Fermi distribution $f(\varepsilon) = 1/(\exp -(\varepsilon - \mu)/k_B T)$, with the chemical potential μ. Draw a sketch of the integrand of this expression as a function of ε for two temperatures, $T = 0$ and $T > 0$, and fixed μ. Using this sketch, discuss qualitatively that ρ must change with increasing temperature, if μ is fixed and $N(\varepsilon)$ is not constant, i.e.. $dN(\varepsilon)/d\varepsilon \neq 0$ near the Fermi level (as in $d = 3$ dimensions).

(b) However, the Coulomb charging energy of the electrons implies local charge neutrality, that is, ρ must be the same everywhere in the wire, especially at its ends,

$$\rho(T_L) = \rho(T_R) \qquad \text{or, for small temperature difference,} \qquad \frac{d\rho(T)}{dT} = 0.$$

Since the chemical potential is not fixed by an outside battery and will adjust itself accordingly, i.e. $\mu = \mu(T)$.

Taking the total temperature derivative of the expression for $\rho(T)$, show that

$$0 \overset{!}{=} \frac{d\rho(T)}{dT} = \int d\varepsilon\, N(\varepsilon) \frac{\varepsilon - \mu}{T} \left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) + \int d\varepsilon\, N(\varepsilon) \left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \frac{\partial \mu}{\partial T}\,.$$

(c) Since $(-\frac{\partial f(\varepsilon)}{\partial \varepsilon})$ is a strongly peaked function, expand $N(\varepsilon) \approx N(\varepsilon_F) + N'(\varepsilon_F)\,(\varepsilon - \mu)$, $N'(\varepsilon_F) = (dN(\varepsilon)/d\varepsilon)|_{\varepsilon_F}$. Consider the symmetry/antisymmetry of the factors in the integrands to derive the thermovoltage, or "thermopower"

$$V_{th} = \frac{1}{e}(\mu(T_R) - \mu(T_L)) = \frac{1}{e}\frac{\partial \mu}{\partial T}\,(T_R - T_L) = -\frac{\pi^2}{6}\frac{N'(\varepsilon_F)}{eN(\varepsilon_F)}\,k_B^2 T\,(T_R - T_L)\,.$$

The thermopower is, thus, controlled by the slope of the density of states at the Fermi level.

(Hint: Substitute $x = (\varepsilon - \mu)/k_B T$ and $\int dx\, x^2(-f'(x)) = \pi^2/6$.)

6.2 Quasiclassical motion in a static electromagnetic field (10 points)

The quasiclassical equations of motion for a quasiparticle read,

$$\frac{d\mathbf{r}}{dt} = \frac{\partial E_{\mathbf{k}\sigma}(\mathbf{r})}{\partial \mathbf{k}}\,, \qquad \frac{d\mathbf{k}}{dt} = -\frac{\partial E_{\mathbf{k}\sigma}(\mathbf{r})}{\partial \mathbf{r}}\,,$$

where $E_{\mathbf{k}\sigma}(\mathbf{r})$ is the (momentum- and position-dependent) energy eigenvalue, i.e., the Hamilton function of the quasiparticle. (Formally, these equations follow from the Ehrenfest theorem.)

We consider static electric and magnetic fields $\mathbf{E} = -\nabla\Phi(\mathbf{r})$, $\mathbf{B} = \nabla \times \mathbf{A}(\mathbf{r})$, with the 4-potential $(\Phi, \mathbf{A})^T$. Usually, the external field $\mathbf{B}$ varies slowly on the scale of the Fermi wavelength and can be considered constant. The vector potential can then be written as $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$. The quasiparticle energy in the electromagnetic field is then

$$E_{\mathbf{k}\sigma}(\Phi(\mathbf{r}), \mathbf{A}(\mathbf{r})) = E_{\mathbf{k}-\frac{e}{c}\mathbf{A}} + e\Phi(\mathbf{r}) - \sigma\mu_B|\mathbf{B}|$$

where $e < 0$ is the charge of the particle and the spin quantization axis was chosen along $\mathbf{B}$, with $\sigma = \pm 1$.

(a) Show that the equation of motion for $\mathbf{k}$ reads (Lorentz and electrostatic forces)

$$\frac{d\mathbf{k}}{dt} = \frac{e}{c}\mathbf{v_k} \times \mathbf{B} + e\mathrm{E}\,,$$

where $\mathbf{v_k} = \partial E_{(\mathbf{k}-\frac{e}{c}\mathbf{A}),\sigma}/\partial \mathbf{k}$ is the group velocity. Why does the spin-dependent term (Zeeman term) not produce any force?

(Hint: It is convenient to calculate the derivatives in cartesian coordinates component by component. Evaluate, in particular, the tensor $(\partial A_i/\partial r_j)$, $i, j = x, y, z$.)

(b) We now consider the case without electric field, $\mathbf{E} = 0$, $\mathbf{B} \neq 0$. Considering the definition of $\mathbf{v_k}$, can the magnetic field induce a change of energy of the quasiparticle? What are then the trajectories that the momentum describes in $k-$space?