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Theoretical Condensed Matter Physics
Exercise 5

(Submission: 30.11.18, discussion: 03./05.12.18)

5.1 Nonequilibrium quasiparticle distribution in a quantum wire (12 points)

An important case of quasiparticles far away from equilibrium occurs in a resistive quantum wire with a voltage V applied between the ends of the wire. Here, the term *quantum wire* refers to the case that the inelastic scattering length (i.e, the average distance a quasiparticle travels before it undergoes an energy-changing scattering event) is longer than the length and thickness of the wire. Thus, in this regime single-particle wave functions at constant energy are well defined. It is, therefore, called the quantum regime of the wire.
The leads attached to the left and right ends of the wire are each one in equilibrium with itself at temperature T and with the chemical potentials $\mu_{L/R} = \varepsilon_F^{(0)} \pm V/2$, respectively. The length L of the wire is much longer than its width, so that spatial variations only occur with respect to the x coordinate along the wire (see Fig. 1).
The Boltzmann equation for the quasiparticle distribution $n_{\mathbf{k}\sigma}$ reads generally,

$$\frac{\partial n_{\mathbf{k}\sigma}}{\partial t} + \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial n_{\mathbf{k}\sigma}}{\partial \mathbf{r}} - \frac{\partial E_{\mathbf{k}\sigma}}{\partial \mathbf{r}} \cdot \frac{\partial n^0_{\mathbf{k}\sigma}}{\partial \mathbf{k}} = I\left\{n_{\mathbf{k}\sigma}\right\}.$$

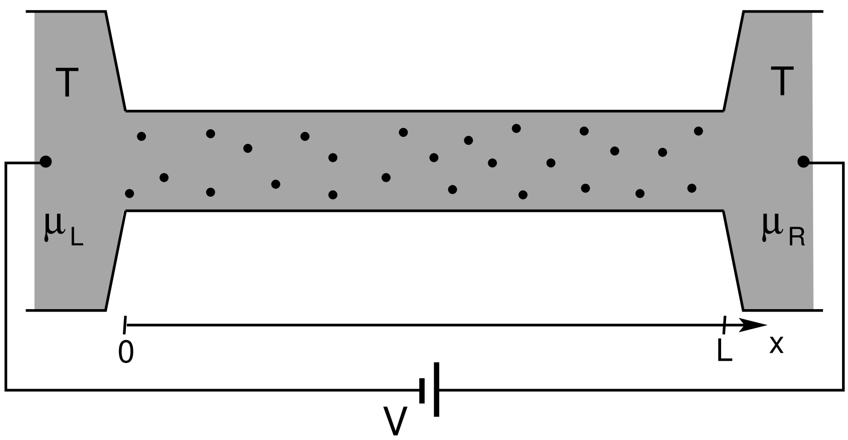

Figure 1: A diffusive quantum wire with applied voltage V.

We now derive the quasiparticle distribution as a function of energy at a position x along the wire in the quantum regime.

(a) Why is in a Fermi liquid the quantum regime (as defined above) always reached for sufficiently low temperature, at least when the applied voltage is zero?

(b) Why does the collision integral vanish in this quantum regime, $I\{n_{\mathbf{k}\sigma}\} = 0$?

(c) Considering the fact that well defined quasiparticle energy states exist in the entire wire (see above), can $E_{\mathbf{k}\sigma}$ be position dependent? What does this imply for the third term on the left-hand side of the Boltzmann equation? Conclude from these considerations what the electrostatic potential inside the wire is. Where in this setup must then the applied voltage V drop?

Now write down the Boltzmann equation for this time-independent situation.

(d) Although there are no inelastic interactions, there is elastic scattering from multiple impurity atoms in the wire. This changes the quasiparticle momenta, leads to diffusive motion of the quasiparticles and ultimately produces the electrical resistivity. We define the quasiparticle density and the quasiparticle current density for particles at a given energy E as,

$$n(E, \mathbf{r}) = \sum_{\mathbf{k}\sigma} n_{\mathbf{k}\sigma}(\mathbf{r})\, \delta(E_{\mathbf{k}\sigma} - E), \qquad \mathbf{j}(E, \mathbf{r}) = \sum_{\mathbf{k}\sigma} \mathbf{v}_F\, n_{\mathbf{k}\sigma}(\mathbf{r})\, \delta(E_{\mathbf{k}\sigma} - E)$$

In a diffusive system, the quasiparticle current density is proportional and opposite in direction to the gradient of the quasiparticle density, $\mathbf{j}(\mathbf{E}) = -D\nabla_{\mathbf{r}} n(E)$, with the diffusion constant D.

Perform an appropriate summation over spin and momentum on the Boltzmann equation to show that in the diffusive regime it reduces to the simple form

$$D\frac{\partial^2}{\partial x^2} n(E, x) = 0.$$

(e) To determine the energy dependent distribution function $n(E, x)$ in the wire, make the ansatz $n(E, x) = A(x)\, n_L^{(0)}(E) + B(x)\, n_R^{(0)}(E)$, where $n_{L/R}^{(0)}(E)$ is the equilibrium Fermi distribution function in left/right lead. The 2nd x-derivative of $n(E, x)$ must vanish for all E. What does this imply for the coefficients $A(x)$, $B(x)$?

Now determine $A(x)$ and $B(x)$ such that the boundary conditions at the left and right ends are fulfilled, $n(E, 0) = n_L^{(0)}(E)$, $n(E, L) = n_R^{(0)}(E)$. Draw $n(e, x)$ as a function of E for several positions x. This non-equilibrium distribution was first predicted and experimentally observed in 1998.

5.2 Sound waves in a Fermi liquid (8 points)

We consider the collision-dominated regime (1st sound): $\omega_s \ll 1/\tau$. Make the ansatz of a time-dependent, local equilibrium distribution

$$\begin{aligned} n_{\mathbf{k}\sigma}(\mathbf{r}, t) &= n_{\mathbf{k}\sigma}^{(0)}(E_{\mathbf{k}\sigma} - \varepsilon_F(\mathbf{r}, t)) \\ \varepsilon_F(\mathbf{r}, t)) &= \varepsilon_F^{(0)} + \Delta\varepsilon_F\, \mathrm{e}^{\mathrm{i}(\mathbf{k}_{s1}\mathbf{r} - \Omega_{s1} t)} \end{aligned}$$

with a complex sound frequency $\Omega_{s1} = \omega_{s1} - i/\tau_{1s}$ (damping!) to solve the collision-less Boltzmann equation

$$\frac{\partial n_{\mathbf{k}\sigma}}{\partial t} + \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial n_{\mathbf{k}\sigma}}{\partial \mathbf{r}} - \frac{\partial E_{\mathbf{k}\sigma}}{\partial \mathbf{r}} \cdot \frac{\partial n_{\mathbf{k}\sigma}^0}{\partial \mathbf{k}} = 0.$$

Why does the collision integral $I\{n_{\mathbf{k}\sigma}\}$ vanish in the collision-dominated regime (see also problem 4.2)? Determine the 1st-sound dispersion $\omega_{1s}(\mathbf{k}) = \mathrm{Re}(\Omega_{1s}(\mathbf{k}))$ and the damping rate $1/\tau_{1s}(\mathbf{k}) = \mathrm{Im}(\Omega_{1s}(\mathbf{k}))$. Note that the solution is damped even though the sollision integral is zero.