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Special Topics in Condensed Matter Theory
Winter term 2016/17

Exercise 7

(Solutions due on 18 January, 2017)

1. The SSH model: band structure (10 points)

Consider the Su-Schrieffer-Heeger (SSH) model, as introduced in the lecture,

$$H = \sum_n \left[\varepsilon_1 \, c^\dagger_{1,n} c_{1,n} + \varepsilon_2 \, c^\dagger_{2,n} c_{2,n} + \left(v \, c^\dagger_{2,n} c_{1,n} + \text{h.c.} \right) + \left(w \, c^\dagger_{1,n+1} c_{2,n} + \text{h.c.} \right) \right] \; .$$

a) Diagonalize the Hamiltonian with respect to the lattice position coordinate n by Fourier transforming to the momentum variable k. Show that the Hamiltonian takes the form,

$$H = \sum_k \begin{pmatrix} c^\dagger_{1,k} & c^\dagger_{2,k} \end{pmatrix} \left[\vec{h}(k) \cdot \vec{\sigma} + \varepsilon_0 \mathbf{1} \right] \begin{pmatrix} c_{1,k} \\ c_{2,k} \end{pmatrix}$$

where $\vec{\sigma}$ is the vector of Pauli matrices and $\mathbf{1}$ the 2×2 unit matrix. Determine the vector $\vec{h}(k)$ and ε_0. What is the range of the momentum values k?

b) Now diagonalize the Hamiltonian completely in the 2-dimensional space of the lattice basis and determine the band structure, $\varepsilon_{1,k}$, $\varepsilon_{2,k}$ as well as the respective eigenvektors.

c) Sketch the dispersions $\varepsilon_{1,k}$, $\varepsilon_{2,k}$ as a function of k for the cases (i) $\varepsilon_1 = \varepsilon_2 = 0$, $v > w$, (ii) $\varepsilon_1 = \varepsilon_2 = 0$, $v = w$, and for (iii) $\varepsilon_1 \neq \varepsilon_2$, $v > w$.

1. The SSH model: topological structure and Chern number (10 points)

a) The Chern number is the generalization of a winding number. Give the general definition of the Chern number C.

b) For the SSH model, calculate the Berry potential $A^n(k)$, $n = 1, 2$, in terms of the components of the vector $\vec{h}(k)$ only, using the results of problem 1.

c) Each of the two bands of the SSH model has its own Chern number. Why? Calculate the Chern number for each of the two bands, C_1, C_2 for the cases (i) $\varepsilon_1 = \varepsilon_2 = 0$, $v > w$ and (ii) $\varepsilon_1 \neq \varepsilon_2$, $v > w$.

d) For the case $\varepsilon_1 = \varepsilon_2 = 0$, discuss how C_1 and C_2 change when the hopping amplitudes change continuously from $v > w$ to $v < w$.