Universität Bonn 16 December, 2016
Physikalisches Institut
Prof. Dr. Johann Kroha, Tim Lappe, Haixin Qiu
http://www.kroha.uni-bonn.de/teaching

Special Topics in Condensed Matter Theory
Winter term 2016/17

Exercise 5

(Solutions due on 23 December, 2016)

1. Dynamical, local correlations in a Fermi sea: Matsubara sum (20 points)

We consider a Fermi sea of non-interacting electrons with the dispersion ε_{vecp} and a lower and upper conduction band edge $\pm D$. For simplicity, we assume that the density of states $N(\varepsilon)$ is constant over the entire band, i.e.,

$$N(\varepsilon) = \frac{1}{2D}\,\Theta(D_0 - |\varepsilon|)\ .$$

Note that the step-like bhavior at the band edges is realized in two-dimensional systems. However dimensionality will not be further considered here.

a) Calculate the *local* R/A electron Green's function $G^{R/A}_{\vec{r}=0}(\omega) = \sum_{\vec{k}} G^{R/A}_{\vec{k}}(\omega)$. In order to perform this task, remember how $\mathrm{Im}G^{R/A}_{\vec{r}=0}(\omega)$ is related to $N(\varepsilon)$. Thus, the momentum summation need not be performed explicitely, and it is not necessary to know the dispersion. Use the Kramers-Kronig relation to compute $\mathrm{Re}G^{R/A}_{0}(\omega)$.

The *spatially local* (i.e., $\vec{r} - \vec{r}' = 0$), retarded density-density correlation function (response function), as a function of time difference $t - t'$, is defined as

$$\chi^R_{\rho\rho}(\vec{r} - \vec{r}' = 0, t - t') = -i\Theta(t - t')\,\langle[\hat{\rho}(\vec{r}, t), \hat{\rho}(\vec{r}, t')]\ \ ,\rangle$$

where $\hat{\rho}(\vec{r}, t) = \Psi^\dagger(\vec{r}, t)\Psi(\vec{r}, t)$ is the electron density operator at an arbitrary position $\vec{r}$.

b) Draw all Feynman diagrams contributing to $\chi_{\rho\rho}(0, t-t')$ and draw for all Green's functions the appropriate space and time labels.
Then draw all Feynman diagrams contributing to $\chi_{\rho\rho}(0, \Omega)$, i.e. the Fourier transform w.r.t. time, and label all Green's functions with the appropriate energies.

c) Write down, for each diagram, the mathematical expression (including the correct sign) in Matsubara representation and perform the Matsubara sum, using contour integration. In this way, calculate first the imaginary part $\mathrm{Im}\chi^R_{\rho\rho}(0, \Omega)$ and the real part.

d) Draw $\mathrm{Im}\chi^R_{\rho\rho}(0, \Omega)$ and $\mathrm{Re}\chi^R_{\rho\rho}(0, \Omega)$ as a function of Ω.