Universität Bonn June 13, 2018
Physikalisches Institut
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Advanced Theoretical Condensed Matter Physics
Summer term 2018

Exercise 5

(Solutions due on June 26, 2018, 12h)

5.1 Renormalization group (RG) for the anisotropic Kondo model (20 points)

The Hamiltonian of the anisotropic Kondo model reads,

$$H = \sum_{\boldsymbol{p}\sigma} \varepsilon_{\boldsymbol{p}} c^\dagger_{\boldsymbol{p}\sigma} c_{\boldsymbol{p}\sigma} + J_{||} \hat{S}_z \, \hat{\sigma}_z + J_\perp \left(\hat{S}_x \hat{\sigma}_x + \hat{S}_y \hat{\sigma}_y \right) \ , \tag{1}$$

where $\hat{S}_i$, $i = x, y, z$, are the spin operators in impurity spin space and $\hat{\sigma}_i = \sum_{\boldsymbol{p}\boldsymbol{p}'\sigma\sigma'} c^\dagger_{\boldsymbol{p}\sigma} \sigma_i c_{\boldsymbol{p}'\sigma'}$, $i = x, y, z$, with the Pauli matrices σ_i. We define the dimensionless coupling constants $g_{||} = N(0) J_{||}$, $g_\perp = N(0) J_\perp$, with $N(0)$ the conduction electron density of states at the Fermi level. In the lecture the perturbative RG equations (to one-loop order) were derived as,

$$\frac{dg_{||}}{d \ln D} = -2 \, {g_\perp}^2 \tag{2}$$

$$\frac{dg_\perp}{d \ln D} = -2 \, g_{||} \, g_\perp \ , \tag{3}$$

where the density of states is assumed to be flat, $N(0) = 1/(2D_0)$, D_0 being half the bare conduction bandwidth (cutoff). We now want to solve these equations for arbitrary (anisotropic) bare couplings $g_{||\,0}$, $g_{\perp\,0}$.

a) From the above RG equations, derive a single differential equation in terms of $dg_{||}/dg_\perp$ which relates $g_{||}$ and $g_\perp$ to each other, and which does not explicitly depend on the cutoff D.

b) Solve the differential equation obtained in problem a) by separation of variables. What type of curves are the solutions $g_\perp(g_{||})$ or $g_{||}(g_\perp)$?

c) Draw the bundles of all possible types of $g_\perp(g_{||})$ trajectories in a $g_\perp$ vs. $g_{||}$ diagram. Note that $g_\perp$ and $g_{||}$ may be positive or negative.

d) Assign to each trajectory an arrow which marks the flow direction of the coupling constants along that trajectory, using the RG equations (2), (3). Using thes flow directions, identify the three different types of low-energy RG fixed points. (Note: $g \to \infty$ is also a fixed point in the RG sense.) Which of these fixed points are stable, which ones are unstable? Discuss which physical ground state each fixed point corresponds to.

e) Knowing $g_\perp(g_{||})$ from problem b), now determine the "running" coupling constant $g_{||}(D)$ from the RG equation (2) as a function of the cutoff D. Determine $g_\perp(D)$ as well.