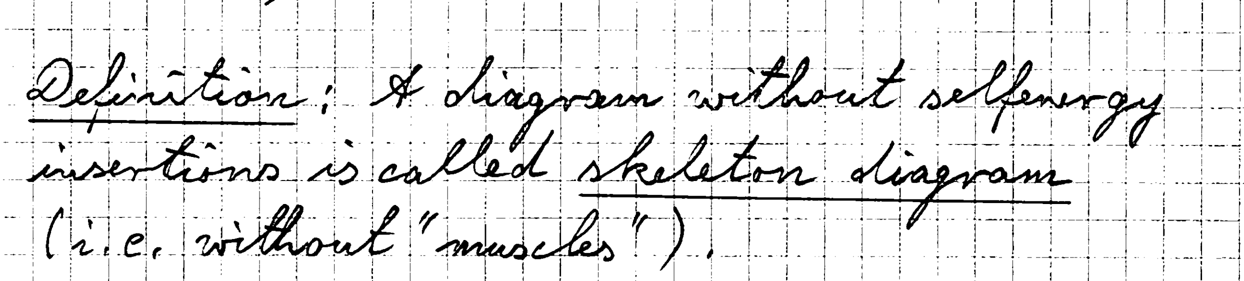
- $\Sigma_p(z)$ determines the position of the poles of $G_p(z)$. It can be shown that it shifts the poles of $G_p^{R/A}(z)$ to the lower / upper half plane only, i.e. <u>causality is preserved</u>, and $\tilde{\varepsilon}_p$, γ, and Z_p are deduced from Σ as in 1.3.2

- The Dyson equation enables us to sum up infinite series of irreducible contributions, while preserving causality.
  It suggests controlled approximations for the irreducible part Σ.

1.4.5 <u>Selfconsistent approximations</u>

The sum of <u>all</u> selfenergy insertions in a diagram can be performed by replacing $G_0 \to G$ everywhere in a diagram, e.g.:



<u>Definition</u>: A diagram without selfenergy insertions is called <u>skeleton diagram</u> (i.e. without "muscles").

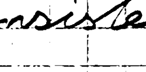
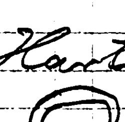
It follows that the selfenergy Σ is the sum of all irreducible skeleton diagrams, with each Green's function line G_0 replaced by the full, renormalized Green's function G.
Hence, the selfenergy is a functional of the full Green's function G:

$$\boxed{\begin{aligned} \Sigma_{\vec{p}}(\omega) &= \Sigma\{G\} \qquad \text{with} \\ G_{\vec{p}}(\omega) &= \frac{1}{\omega+\mu-\varepsilon_{p}-\Sigma_{\vec{p}}(\omega)} \end{aligned}}$$

Since the selfenergy involves calculating the full G-function, which in turn requires the knowledge of Σ, this is a <u>selfconsistent</u> set of equations. It is usually solved by iteration.

Selfconsistent approximations are generated by defining an approximate functional $\Sigma\{G\}$, e.g. by taking a finite order of skeleton diagrams.

<u>Example:</u> selfconsistent Hartree-Fock:

$\Sigma =$  $+$  1st order in V.

<u>Remark</u> (without proof):

How is the approximate $\Sigma\{G\}$ to be chosen in an "optimal" way?
So-called conserving approximations are generated by deriving Σ from a generating functional $\Phi\{G\}$ by functional derivative:

$$\Sigma\{G\} = \frac{\delta \Phi\{G\}}{\delta G}$$

Where Φ is a sum of closed skeleton diagrams.