In this expression $\hat{T}$ places each term of $S(\beta)$ in the correct place between $\psi(\tau)$, $\psi^+(0)$, and the multiplication rule for $S(\tau_2, \tau_1)$ has been used.

The perturbation theory is now developed as an expansion in powers of V_I. This requires to evaluate expectation values of higher-order products of field operators with equal numbers of ψ and of ψ^+.
This is substantially facilitated by a factorization rule, Wick's theorem.

1.4.2 Wick's theorem

We define the <u>many-particle Green's function</u> for a non-interacting system as

$$G_n^0(\tau_1, \dots, \tau_n; \tau_1', \dots, \tau_n') =$$
$$= -\frac{1}{Z_G^0} \, tr\left\{ e^{-(H_0 - \mu N)\beta} \hat{T}\left\{ \psi(\tau_1) \dots \psi(\tau_n) \psi^+(\tau_1') \dots \psi^+(\tau_n') \right\}\right\}$$

where all operators are in the interaction picture:

$$\psi(\tau_i) = e^{(H_0 - \mu N)\tau_i} \psi \, e^{-(H_0 - \mu N)\tau_i} \quad \text{etc.}$$

The factorization of the tr of operator products is derived using the equation of motion for G:

The EOM for the thermal single-particle Green's fct. is

$$\left[-\frac{\partial}{\partial\tau} - (H_0 - \mu N)_\tau\right] G(\tau, \tau') = \delta(\tau - \tau')$$

← acting on the operator at time τ

The EOM for the non-interacting n-particle Green's function is then obtained as

$$\left[-\frac{\partial}{\partial\tau_i} - (H_0 - \mu N)_{\tau_i}\right] G_n^0(\tau_1, \ldots, \tau_n; \tau_1', \ldots, \tau_n')$$

$$= \sum_{j=1}^{n} (\underset{B}{\overset{F}{\mp}} 1)^{j-1+n-i} \delta(\tau_i - \tau_j') G_{n-1}^0(\tau_1, \ldots, \cancel{\tau_i}, \ldots \tau_n; \tau_1', \ldots, \cancel{\tau_j'}, \ldots, \tau_n')$$

number of commutations needed to bring $\Psi^+(\tau_j')$ and $\Psi(\tau_i)$ together.

$$G_n^0(\tau_1, \ldots \tau_n, \tau_1', \ldots \tau_n') \underset{\text{S.P. EOM}}{=} \sum_{j=1}^{n} (\mp 1)^{j-1+n-i} \quad G(\tau_i, \tau_j') G_{n-1}^0(\tau_1 \ldots \cancel{\tau_i} \ldots \tau_n; \tau_1' \ldots \cancel{\tau_j'} \ldots \tau_n')$$

By induction it follows <u>Wick's theorem</u>:

> The non-interacting n-particle Green's function (of operators with canonical commutation relations!) factorizes into a sum over products of free 1-particle Green's functions $G_1^0(\tau_i, \tau_j')$
> The sum runs over all possible pairings of operators $\Psi(\tau_i)$, $\Psi^+(\tau_j')$
> For each pair there is a factor $(\mp 1)^P$ (fermions / bosons) where P is the number of elementary commutations required to bring the respective operators together.