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Physikalisches Institut
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Equilibrium and Non-equilibrium QFT in Many-Body Systems SS 2013

Exercise Sheet 6

(To be discussed on the 18th and 19th of July)

6.1 Relationships between Green's Functions in the Keldysh Formalism

In this exercise we will derive the relations between the retarded and advanceed Green's functions G^R and G^A and the quantities $iG^{--}(\mathbf{x}_1, \mathbf{x}_2)$, $iG^{++}(\mathbf{x}_1, \mathbf{x}_2)$, $iG^{+-}(\mathbf{x}_1, \mathbf{x}_2)$ and $iG^{-+}(\mathbf{x}_1, \mathbf{x}_2)$ in the Schwinger-Keldysh space, as defined in the lecture. Here $\mathbf{x}_i \equiv (t_i, \vec{r}_i)$.
Using the definitions given in the lecture, show that the retarded and advanced Green's functions G^R and G^A fulfills:

(i) $G^R(\mathbf{x}_1, \mathbf{x}_2) = G^{--}(\mathbf{x}_1, \mathbf{x}_2) - G^{-+}(\mathbf{x}_1, \mathbf{x}_2) = G^{+-}(\mathbf{x}_1, \mathbf{x}_2) - G^{++}(\mathbf{x}_1, \mathbf{x}_2)$.

(ii) $G^A(\mathbf{x}_1, \mathbf{x}_2) = G^{--}(\mathbf{x}_1, \mathbf{x}_2) - G^{+-}(\mathbf{x}_1, \mathbf{x}_2) = G^{-+}(\mathbf{x}_1, \mathbf{x}_2) - G^{++}(\mathbf{x}_1, \mathbf{x}_2)$.

Hint: $G^{--}(\mathbf{x}_1, \mathbf{x}_2) + G^{++}(\mathbf{x}_1, \mathbf{x}_2) = G^{-+}(\mathbf{x}_1, \mathbf{x}_2) + G^{+-}(\mathbf{x}_1, \mathbf{x}_2)$

6.2 Relationships between Self-Energies in the Keldysh Formalism

Show that the self-energy components in the Schwinger-Keldysh representattion Σ^{++}, Σ^{--}, Σ^{+-} and Σ^{-+} are related to one another via the relation

$$\Sigma^{++} + \Sigma^{--} = -\left(\Sigma^{+-} + \Sigma^{-+}\right).$$

To show this, write down the Dyson equation in the Schwinger-Keldysh representation. Act on this equation from the left by the operator $\hat{G}^{-1}(\mathbf{x}_0, \mathbf{x}_1)$, defined in the lecture as

$$\hat{G}^{-1}(\mathbf{x}_0, \mathbf{x}_1) \equiv i\delta(t_0 - t_1)\left[\frac{d}{dt} + \mu - H_0\right].$$

Note that $G^{-1}(\mathbf{x}_0, \mathbf{x}_1)$ acts on the free Keldysh matrix Green's function as

$$G^{-1}(\mathbf{x}_0, \mathbf{x}_1)G^{(0)}(\mathbf{x}_1, \mathbf{x}_2) = \sigma_z \delta(\mathbf{x}_0 - \mathbf{x}_2).$$

Then use the relation for the Green's functions

$$G^{--} + G^{++} - G^{-+} - G^{+-} = 0,$$

from the lecture.