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Physikalisches Institut
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http://www.kroha.uni-bonn.de/teaching/lectures-and-seminars/ss13/cmft

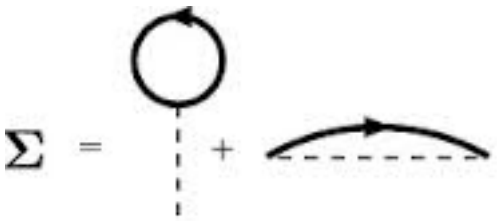
Equilibrium and Non-equilibrium QFT in Many-Body Systems SS 2013

Exercise Sheet 5

(To be discussed on the 4th and 5th of July)

5.1 Hartree-Fock Diagrams

Using the Feynman rules you have learned from the lecture, evaluate the following *irreducible* self-energy diagrams in the Matsubara formalism, where the dotted lines denote the interaction $V_{\mathbf{q}}$, and



the solid lines are the free Green's functions in the Matsubara formalism.

5.2 2nd-Order Perturbation Theory in High Dimensions

The summation over internal momenta in any evaluation of Feynman diagrams in perturbation theory is usually quite difficult to do. However, it has been shown that in very high spatial dimensions, a simplification occurs in that one can replace, in an arbitrary diagram, all momentum-dependent Green's functions $G_{\mathbf{k}\sigma}(i\omega_n)$ with *local* ones $G_{0\sigma}(i\omega_n) \equiv \sum_{\mathbf{k}} G_{\mathbf{k}\sigma}(i\omega_n)$. In other words, there is no momentum conservation at the vertices of diagrams in this limit, although full frequency conservation at the vertex is still preserved.
We consider the following diagram:

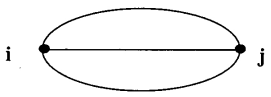


Figure 1: 2nd Order Irreducible Self-Energy Diagram for the Hubbard Model

Fig. 1 is the irreducible self-energy diagram of second order in the interaction vertices for the *Hubbard model*, in which the interaction U is momentum-independent. All lines correspond to *noninteracting* Green's functions, and the filled dots correspond to the interaction vertices with strength U.

a) Write down the expression corresponding to the diagram in Fig. 1 in terms of the noninteracting Green's functions and summations over internal momenta and Matsubara frequencies. Do this according to the *usual* Feynman rules.

b) Consider the expression you obtain in *a*). Now replace the summation over internal momenta with individual summations over the respective Greens' function momentum, such that you obtain a product of *local* Green's functions. Evaluate the resulting expression via the usual Matsubara summations.